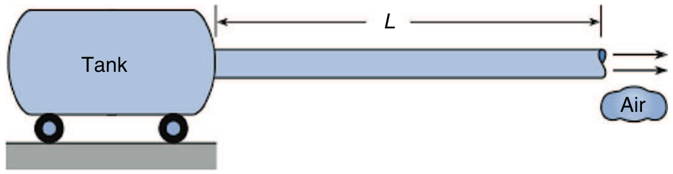
10.6: PROBLEM DEFINITION

Situation:

SITUATION

Air is flowing from a large tank to ambient through a horizontal pipe.
Pipe is 25 mm Schedule 40. $D = 0.0266\,\mathrm{m}$.
$V = 10\,\mathrm{m/s}$. $f = 0.015$, $L = 50\,\mathrm{m}$.



Assumptions:
Air has constant density (look up properties at 1 atm).
KE correction factor is $\alpha_2 = 1.0$.

Properties:
Air (20 °C, 1 atm, Table A.3): $\rho = 1.2\,\mathrm{kg/m^3}$.

PLAN

1. Relate pressure in tank to head loss using the energy equation.
2. Describe head loss using the Darcy-Weisbach equation.
3. Combine steps 1 & 2.

SOLUTION

1. Energy eqn. (location 1 inside the tank, location 2 at the pipe exit)

$$\begin{aligned}\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ \frac{p_1}{\gamma} + 0 + 0 + 0 &= 0 + \frac{V_2^2}{2g} + 0 + h_f \qquad (1)\end{aligned}$$

2. Darcy-Weisbach eqn.:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \qquad (2)$$

3. Combine Eqs. (1) and (2).

$$\begin{aligned}p_1 &= \gamma \frac{V_2^2}{2g}\left(1 + f\frac{L}{D}\right) = \frac{\rho V_2^2}{2}\left(1 + f\frac{L}{D}\right) \\ &= \frac{(1.2\,\mathrm{kg/m^3})\,(10\,\mathrm{m/s})^2}{2}\left(1 + (0.015)\,\frac{(50\,\mathrm{m})}{(0.0266\,\mathrm{m})}\right) \\ &= 1.75\,\mathrm{kPa\text{-}gage}\end{aligned}$$

$$\boxed{p_{\text{tank}} = 1.75\,\text{kPa gage}}$$

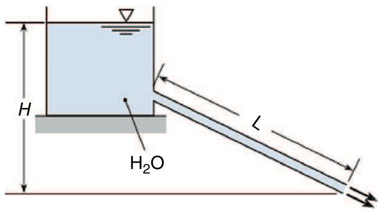
REVIEW The constant density assumption is valid because the pressure in the tank is less than 2% of atmospheric pressure.

10.8: PROBLEM DEFINITION

Situation:

Water is flowing from a tank through a tube & then discharging to ambient.
$D = 0.008\,\text{m}$, $L = 6\,\text{m}$.
$H = 3\,\text{m}$, $f = 0.015$.

Sketch:



Find:

Exit velocity (m/s).
Discharge (L/s).
Sketch the HGL & EGL.

Assumptions:

The only head loss is in the tube.
Turbulent flow so $\alpha_2 = 1.0$.

Properties:

Water (15 °C), Table A.5 (EFM10e), $\rho = 999\,\text{kg/m}^3$ $\nu = 1.14 \times 10^{-6}\,\text{m}^2/\text{s}$.

PLAN

1. Relate H to head loss using the energy equation.
2. Describe head loss using the Darcy-Weisbach equation.
3. Find V by combining steps 1 & 2.
4. Find Q by using the flow rate equation.

SOLUTION

1. Energy eqn. (location 1 at the free surface, location 2 at the pipe exit)

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

$$0 + 0 + H + 0 = 0 + \frac{V_2^2}{2g} + 0 + h_L \tag{1}$$

2. Darcy-Weisbach eqn.:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \tag{2}$$

3. Combine Eqs. (1) and (2):

$$H = \frac{V_2^2}{2g}\left(1 + f\frac{L}{D}\right)$$

$$V_2 = \sqrt{\frac{2gH}{1 + f\frac{L}{D}}}$$

$$= \sqrt{\frac{2\,(9.81\,\mathrm{m/\,s^2})\,(3\,\mathrm{m})}{1 + 0.015\frac{(6\,\mathrm{m})}{(0.008\,\mathrm{m})}}}$$

$$\boxed{V = 2.19\,\mathrm{m/\,s}}$$

4. Flow rate equation:

$$Q = VA = V\frac{\pi D^2}{4} = (2.192\,\mathrm{m/\,s})\,\frac{\pi\,(0.008\,\mathrm{m})^2}{4}$$

$$\boxed{Q = 0.110\ \mathrm{L/\,s}}$$

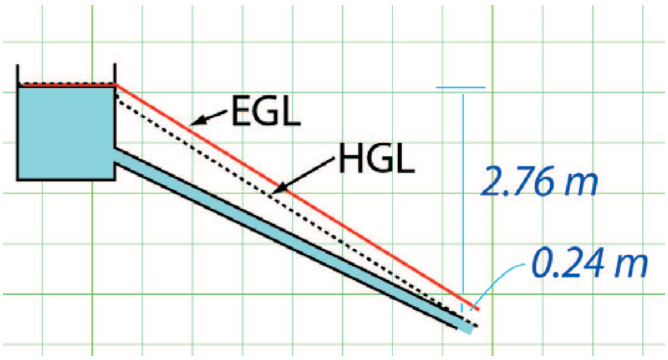
5. Sketch HGL & EGL

- Locate the EGL & HGL on free surface of tank.
- Velocity head and head loss:

$$\frac{V^2}{2g} = \frac{(2.192\,\mathrm{m/\,s})^2}{2\,(9.81\,\mathrm{m/\,s^2})} = 0.24\,\mathrm{m}$$

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = 0.015\frac{(6\,\mathrm{m})}{(0.008\,\mathrm{m})}\frac{(2.192\,\mathrm{m/\,s})^2}{2\,(9.81\,\mathrm{m/\,s^2})} = 2.76\,\mathrm{m}$$

- Locate the EGL and HGL at the end of the pipe. Sketch lines.



REVIEW Check the turbulent flow assumption.

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(2.192\,\mathrm{m/\,s})\,(0.008\,\mathrm{m})}{(1.14\times10^{-6}\,\mathrm{m^2/\,s})}$$

$$\mathrm{Re} = 15400 > 3000$$

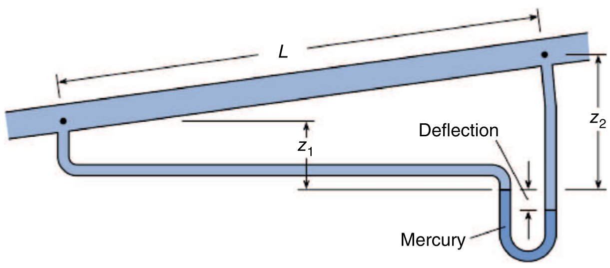
Thus, the assumption of turbulent flow is valid.

10.27: PROBLEM DEFINITION

Situation:

Oil flows through a smooth pipe.
$L = 12\,\text{m}$, $z_1 = 1\,\text{m}$, $z_2 = 2\,\text{m}$.
$V = 1.2\,\text{m/s}$, $D = 5\,\text{cm}$.

Sketch:



Find:

Flow direction.
Resistance coefficient.
Nature of flow (laminar or turbulent).
Viscosity of oil (N s/m^2).

Properties:

$S_{oil} = 0.8$; $S_{Hg} = 13.6$.

SOLUTION

Based on the deflection on the manometer, the piezometric head (and HGL) on the right side of the pipe is larger than that on the left side.
Thus, the **flow is downward (from right to left).**

Energy principle

$$\frac{p_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + z_1 + h_L$$

Assume $\alpha_1 V_1 = \alpha_2 V_2$. Let $z_2 - z_1 = 1\,\text{m}$. Also the head loss is given by the Darcy Weisbach equation: $h_f = f(L/D)V^2/(2g)$. The energy principle becomes

$$\frac{p_2 - p_1}{\gamma_{\text{oil}}} = (-1\,\text{m}) + f\frac{L}{D}\frac{V^2}{2g} \tag{1}$$

Manometer equation

$$p_2 + (2\,\text{m})\,\gamma_{\text{oil}} + (0.1\,\text{m})\,\gamma_{\text{oil}} - (0.1\,\text{m})\,\gamma_{\text{Hg}} - (1\,\text{m})\,\gamma_{\text{oil}} = p_1$$

Algebra gives

$$
\begin{aligned}
\frac{p_2 - p_1}{\gamma_{\text{oil}}} &= -(2\,\text{m}) - (0.1\,\text{m}) + (0.1\,\text{m})\frac{\gamma_{\text{Hg}}}{\gamma_{\text{oil}}} + (1\,\text{m}) \\
&= -(1\,\text{m}) + (0.1\,\text{m})\left(\frac{S_{\text{Hg}}}{S_{\text{oil}}} - 1\right) \\
&= -(1\,\text{m}) + (0.1\,\text{m})\left(\frac{13.6}{0.8} - 1\right) \\
\frac{p_2 - p_1}{\gamma_{\text{oil}}} &= 0.6\,\text{m} \qquad (2)
\end{aligned}
$$

Substituting Eq. (2) into (1) gives

$$
\begin{aligned}
(0.6\,\text{m}) &= (-1\,\text{m}) + f\frac{L}{D}\frac{V^2}{2g} \\
&\text{or} \\
f &= 1.6\left(\frac{D}{L}\right)\left(\frac{2g}{V^2}\right) \\
&= 1.6\left(\frac{0.05\,\text{m}}{12\,\text{m}}\right)\left(\frac{2 \times 9.81\,\text{m/s}^2}{(1.2\,\text{m/s})^2}\right) \\
&\boxed{f = 0.0908}
\end{aligned}
$$

Since the resistance coefficient is now known, this value can be used to find viscosity. To perform this calculation, assume the flow is laminar.

$$
\begin{aligned}
f &= \frac{64}{\text{Re}} \\
0.0908 &= \frac{64\mu}{\rho V D} \\
&\text{or} \\
\mu &= \frac{0.0908 \rho V D}{64} \\
&= \frac{0.0908 \times (0.8 \times 1000) \times 1.2 \times 0.05}{64} \\
&\boxed{\mu = 0.068\,\text{N} \cdot \text{s/m}^2}
\end{aligned}
$$

Now, check Reynolds number to see if laminar flow assumption is valid

$$
\begin{aligned}
\text{Re} &= \frac{V D \rho}{\mu} \\
&= \frac{1.2 \times 0.05 \times (0.8 \times 1000)}{0.068} \\
&= 706
\end{aligned}
$$

Thus, flow is $\boxed{\text{laminar.}}$

10.37: PROBLEM DEFINITION

Situation:

Water flows in a cast iron pipe.
$D = 0.16\,\text{m}$, $Q = 0.1\,\text{m}^3/\,\text{s}$.
$k_s = 0.26\,\text{mm}$.

Find:

Reynolds number.
Friction factor, f.
Shear stress at the wall (Pa).

Properties:

Water (20 °C), Table A.5: $\rho = 998\,\text{kg}/\,\text{m}^3$, $\nu = 1.00 \times 10^{-6}\,\text{m}^2/\,\text{s}$.

SOLUTION

1. Flow rate eqn.

$$\begin{aligned} V &= \frac{Q}{A} = \frac{(0.1\,\text{m}^3/\,\text{s})}{(\pi/4)\,(0.16\,\text{m})^2} \\ &= 4.974\,\text{m}/\,\text{s} \end{aligned}$$

2. Reynolds number

$$\text{Re} = \frac{VD}{\nu} = \frac{(4.974\,\text{m}/\,\text{s})\,(0.16\,\text{m})}{(1.00 \times 10^{-6}\,\text{m}^2/\,\text{s})} = 7.96 \times 10^5$$

$$\boxed{\text{Re} = 7.96 \times 10^5}$$

3. Relative roughness

$$\frac{k_s}{D} = \frac{0.26\,\text{mm}}{160\,\text{mm}} = 1.625 \times 10^{-3}$$

4. Swamee Jain eqn.

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10}\left(\frac{1.625\times10^{-3}}{3.7} + \frac{5.74}{(7.96\times10^5)^{0.9}}\right)\right]^2} = 0.0225 \end{aligned}$$

$$\boxed{f = 0.0225}$$

5. Definition of f:

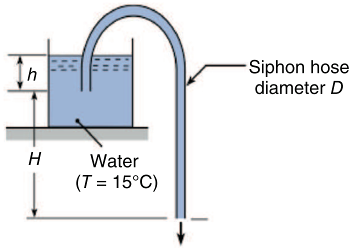
$$\begin{aligned} \tau_0 &= \frac{f\rho V^2}{8} = \frac{0.023\,(998\,\text{kg}/\,\text{m}^3)\,(4.974\,\text{m}/\,\text{s})^2}{8} \\ &= 69.55\,\text{Pa} \end{aligned}$$

$$\boxed{\tau_0 = 69.6\,\text{Pa}}$$

10.53: PROBLEM DEFINITION

Situation:

Water is flowing out a plastic siphon hose.
$D = 0.012\,\text{m}$, $H = 3\,\text{m}$.
$h = 1.0\,\text{m}$, $L = 5.5\,\text{m}$.
$k_s = 0$.



Find:

Velocity (assume the Bernoulli equation applies).
Velocity (include the head loss in the hose).

Assumptions:

Steady flow.
Neglect all head loss (part 1 of problem).
Neglect component head loss (part 2 of problem).
Turbulent flow. Also, $\alpha_2 = 1.0$.

Properties:

Water (15 °C), Table A.5: $\nu = 1.14 \times 10^{-6}\,\text{m}^2/\text{s}$.

PLAN

1. Use the Bernoulli equation to find velocity
Classify this problem as case 2 (V is unknown), then
2. Write the energy eqn., the Darcy-Weisbach eqn., etc. to produce a set of 4 equations with 4 unknowns.
3. Solve the set of equations using a computer program (we used TK Solver).

SOLUTION

1. Bernoulli equation (point 1 on tank surface; point 2 on exit plane of hose):

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

$$0 + (H + h) + 0 = 0 + 0 + \frac{V_2^2}{2g}$$

$$V = \sqrt{2g\,(H+h)} = \sqrt{2\,(9.81\,\text{m/s}^2)\,(3\,\text{m} + 1\,\text{m})}$$

$$\boxed{V = 8.86\,\text{m/s}}$$

2. Equations for finding velocity:

- Energy equation:

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ 0 + 0 + (H + h) + 0 &= 0 + \frac{V_2^2}{2g} + 0 + h_f \qquad (1) \end{aligned}$$

- Darcy-Weisbach:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \qquad (2)$$

- Swamee-Jain:

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \qquad (3)$$

- Reynolds number:

$$\mathrm{Re} = \frac{VD}{\nu} \qquad (4)$$

3. Solution of Eqs. (1) to (4): (Using iterative methods)

$$\begin{aligned} h_f &= 3.66\,\mathrm{m} \\ \mathrm{Re} &= 26930 \\ f &= 0.024 \end{aligned}$$

$$\boxed{V = 2.56\,\mathrm{m/s}}$$

REVIEW

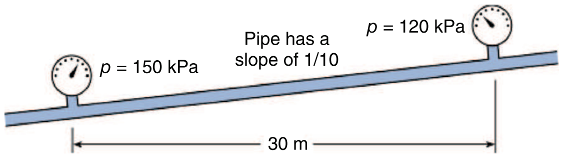
1. Notice that the turbulent flow assumption is valid because Re > 2300.
2. An easy way to solve case 2 and case 3 problems is to acquire a computer program that can solve coupled, non-linear equations.

10.58: PROBLEM DEFINITION

Situation:

A fluid flows through a galvanized iron pipe.
$D = 8\,\text{cm}$.
Pipe slope is 1 Horizontal to 10 Vertical.

Sketch:



Find:

Flow rate.

Properties:

From Table 10.4 $k_s = 0.15$ mm.
$\rho = 800\,\text{kg/m}^3$, $\nu = 10^{-6}\,\text{m}^2/\text{s}$.

Assumptions:

$\alpha_1 = \alpha_2 = 1.0$

SOLUTION

Energy equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_f \\
\frac{150000\,\text{Pa}}{800\,\text{kg/m}^3 \times 9.81\,\text{m/s}^2} + \frac{V_1^2}{2g} + 0 &= \frac{120000\,\text{Pa}}{800\,\text{kg/m}^3 \times 9.81\,\text{m/s}^2} + \frac{V_2^2}{2g} + 3\,\text{m} + h_f \\
h_f &= 0.823\,\text{m} \\
((D^{3/2})/(\nu)) \times (2gh_f/L)^{1/2} &= ((0.08)^{3/2}/10^{-6}) \times (2 \times 9.81 \times 0.823/30.14)^{1/2} \\
&= 1.66 \times 10^4
\end{aligned}$$

Relative roughness

$$\frac{k_s}{D} = \frac{1.5 \times 10^{-4}}{0.08} = 1.9 \times 10^{-3}$$

Resistance coefficient. From Fig. 10.14 (in 10e) $f = 0.025$. Then

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

Solving for V

$$
\begin{aligned}
V &= \sqrt{\left(\frac{h_f}{f}\right)\left(\frac{D}{L}\right)2g} \\
&= \sqrt{\left(\frac{0.823\,\mathrm{m}}{0.025}\right)\left(\frac{0.08\,\mathrm{m}}{30.14\,\mathrm{m}}\right)\times 2\times 9.81\,\mathrm{m/\,s^2}} = 1.312\ \mathrm{m/s} \\
Q &= VA \\
&= 1.312\,\mathrm{m/\,s}\times(\pi/4)\times(0.08\,\mathrm{m})^2 = 6.59\times 10^{-3}\,\mathrm{m^3/\,s}
\end{aligned}
$$

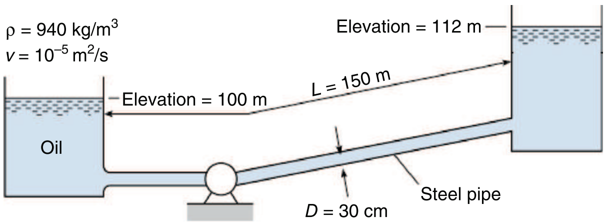
$$
\boxed{Q = 6.59\times 10^{-3}\,\mathrm{m^3/\,s}}
$$

10.69: PROBLEM DEFINITION

Situation:

Oil is pumped from a lower reservoir to an upper reservoir through a steel pipe.
$D = 30\,\text{cm}$, $Q = 0.20\,\text{m}^3/\,\text{s}$.
$z_1 = 100\,\text{m}$, $z_2 = 112\,\text{m}$, $L = 150\,\text{m}$.

Sketch:



Find:

(a) Pump power.
(b) Sketch an EGL and HGL.

Properties:

$\rho = 940\,\text{kg}/\,\text{m}^3$, $\nu = 10^{-5}\,\text{m}^2/\,\text{s}$.
From Table 10.4 $k_s = 0.046$ mm

PLAN

Apply the energy equation between reservoir surfaces .

SOLUTION

Energy equation

$$\begin{aligned}\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \sum h_L \\ 100 + h_p &= 112 + \frac{V^2}{2g}(K_e + f\frac{L}{D} + K_E) \\ h_p &= 12 + \left(\frac{V^2}{2g}\right)\left(0.03 + f\frac{L}{D} + 1\right)\end{aligned}$$

Flow rate equation

$$\begin{aligned}V &= \frac{Q}{A} \\ &= \frac{0.2\,\text{m}^3/\,\text{s}}{(\pi/4) \times (0.30\,\text{m})^2} \\ &= 2.83\ \text{m/s} \\ \frac{V^2}{2g} &= 0.408\,\text{m}\end{aligned}$$

Reynolds number

$$\begin{aligned}\mathrm{Re} &= \frac{VD}{\nu} \\ &= \frac{2.83\,\mathrm{m/\,s} \times 0.30\,\mathrm{m}}{10^{-5}\,\mathrm{m^2/\,s}} \\ &= 8.5 \times 10^4 \\ \frac{k_s}{D} &= \frac{4.6 \times 10^{-5}\,\mathrm{m}}{0.3\,\mathrm{m}} \\ &= 1.5 \times 10^{-4}\end{aligned}$$

Resistance coefficient (from the Moody diagram, Fig. 10.14 in EFM10e)

$$f = 0.019$$

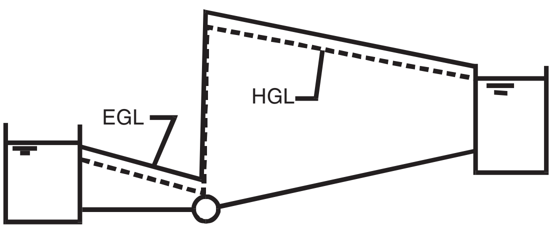
Then

$$\begin{aligned}h_p &= 12\,\mathrm{m} + 0.408\,\mathrm{m}(0.03 + (0.019 \times \frac{150\,\mathrm{m}}{0.3\,\mathrm{m}}) + 1.0) \\ &= 16.3\ \mathrm{m}\end{aligned}$$

Power equation

$$\begin{aligned}P &= Q\gamma h_p \\ &= 0.20\,\mathrm{m^3/\,s} \times (940\,\mathrm{kg/\,m^3} \times 9.81\,\mathrm{m/\,s^2}) \times 16.3\,\mathrm{m} = 30100\,\mathrm{W}\end{aligned}$$

$$\boxed{P = 30.1\,\mathrm{kW}}$$

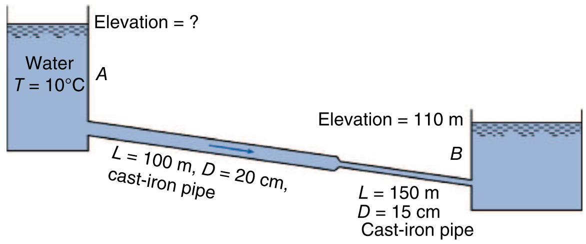


10.85: PROBLEM DEFINITION

Situation:

Two reservoirs are connected by a cast-iron pipe of varying diameter.
$z_2 = 110\,\text{m}$, $Q = 0.3\,\text{m}^3/\,\text{s}$.
$D_1 = 20\,\text{cm}$, $L_1 = 100\,\text{m}$.
$D_2 = 15\,\text{cm}$, $L_2 = 150\,\text{m}$.

Sketch:



Find:

Water surface elevation in reservoir A.

Properties:

From Table 10.4: $k_s = 0.26$ mm.
Water (10 °C), Table A.5: $\nu = 1.3 \times 10^{-6}$ m^2/s.

SOLUTION

$$\frac{k_s}{D_{20}} = \frac{0.26}{200} = 0.0013$$

$$\frac{k_s}{D_{15}} = 0.0017$$

$$V_{20} = \frac{Q}{A_{20}} = \frac{0.03\,\text{m}^3/\,\text{s}}{\pi/4 \times (0.20\,\text{m})^2} = 0.955 \text{ m/s}$$

$$\frac{Q}{A_{15}} = 1.697 \text{ m/s}$$

$$\text{Re}_{20} = \frac{VD}{\nu} = \frac{0.955\,\text{m}/\,\text{s} \times 0.2\,\text{m}}{1.3 \times 10^{-6}\,\text{m}^2/\,\text{s}} = 1.5 \times 10^5$$

$$\text{Re}_{15} = \frac{1.697\,\text{m}/\,\text{s} \times 0.15\,\text{m}}{1.3 \times 10^{-6}\,\text{m}^2/\,\text{s}} = 1.9 \times 10^5$$

From Fig. 10.14 (EFM 10e): $f_{20} = 0.022$; $f_{15} = 0.024$

$$
\begin{aligned}
z_1 &= z_2 + \sum h_L \\
z_1 &= 110 + \frac{V_{20}^2}{2g}\left(0.5 + 0.022 \times \frac{100\,\text{m}}{0.2\,\text{m}} + \sout{0.19}\right) \\
&\quad + \frac{V_{15}^2}{2g}\left[\left(0.024 \times \frac{150\,\text{m}}{0.15\,\text{m}}\right) + 1.0 + 0.19\right] \\
&= 110\,\text{m} + 0.0465\,\text{m}(11.7) + 0.1468\,\text{m}(25.19) \\
&= 110 + 0.535 + 3.70 = 114.2\,\text{m}
\end{aligned}
$$

$$\boxed{z_1 = 114\,\text{m}}$$

0.06

$$= 110\,\text{m} + 0.0465\,\text{m}\,(11.5) + 0.1468\,\text{m}\,(25.06)$$

$$= 110 + 0.535 + 3.679 = \boxed{114.2\,\text{m}}$$

$$\boxed{z_1 = 114.2\ \text{m}}$$